Problem 1

Construct a family of directed graphs such that the random-walk algorithm takes exponential time in expectation to solve the PATH problem on these instances.

Problem 2

A directed graph is called strongly connected if for every pair of vertices, there exists a directed paths between them (in both directions).

Show that the PATH problem reduces in logarithmic space to the problem of deciding strong connectivity.

Problem 3

Show that there exists a polynomial-space algorithm for the following problem (from the previous homework)

\[ L = \{ \varphi \mid \forall x. \exists y. \varphi(x, y) = 1 \} \]

(Here, \( \varphi \) is a Boolean formula in variables \( x_1, x_2, \ldots \) and \( y_1, y_2, \ldots \))