Problem 1

Let $M$ be any non-deterministic finite automaton. Let $M'$ be the automaton obtained from $M$ by adding an $\varepsilon$-transition from each accept state to the start state and making the start state an accept state. Prove or give a counterexample to the statement $L(M') = L(M)^*$. 

Problem 2

Give a regular expression for each of the following languages — full proofs are not necessary:

a. The set of all binary strings that start with 0, end with 1, and have at most three 1’s.

b. The set of all binary that have an odd number of 1’s and contain 00 as a substring.

c. The set of all binary strings that do not contain the substring 001.
Problem 3

Give a non-deterministic finite automaton for each of the following regular expressions over the alphabet \{0, 1\} — full proofs are not necessary:

a. \(0(011)^* \cup 1\).
b. \(0^* \cup 01(01)^*\).
c. \((0 \cup 11^*)00^*11^*\).

Problem 4

For each of the following languages, prove or disprove that the language is regular:

a. \(\{0^s1^t2^{\max\{s, t\}} \mid \text{integers } s, t \geq 0\} \subseteq \{0, 1, 2\}^*\).
b. \(\{0^s1^{2t} \mid \text{integers } s, t \geq 0\} \subseteq \{0, 1\}^*\).
c. \(\{0^s1^{2t}2^{3t} \mid \text{integers } s, t \geq 0\} \subseteq \{0, 1, 2\}^*\).